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Semi-probabilistic design of rainwater tanks: a case study in Northern Italy

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ABSTRACT
The paper proposes a semi-probabilistic approach for the design of rainwater tanks. In particular, the cumulative distribution function of the active storage is derived as a function of rainfall moments. The model is validated through continuous simulation of the hydraulic behaviour of a hypothetical rainwater tank located in Milan (North Italy) using as input a ~~measured~~ series of ~~rainfalls in that area.~~

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1. Introduction

In the last decades, climate change has caused a growth of the number and intensity of extreme events, such as aridity, droughts, heat waves, floods and stormy rainfalls (Pandey *et al.* 2003). Furthermore, the extent of impermeable surfaces has increased, as a consequence of extensive urbanization. In this context, floods and overloads of drainage systems and of treatment plants have become more frequent.

On the contrary, in some developing countries, water scarcity is a major problem: many countries of Africa, Asia and South America have been classified as water-scarce, characterized by low erratic rainfalls, high risk of droughts and intra-seasonal dry spells (Helmreich and Horn 2009). Since rainfall is the most directly accessible water supply source (Su *et al.* 2009), in recent years small on-site Rain Water Harvesting Systems (RWHSs) have been successfully implemented as alternative water supply sources in some countries. These systems intercept rainwater in the hydrologic cycle through either natural landforms or artificial facilities and they have been used to provide supplementary water supplies; rainwater collected from roofs has usually a low content of pollutants, and can be used with low or no treatment for WC flushes, washing machines, irrigation, fire suppression, street washing, etc. Furthermore, these systems can serve as a major water supply source in some rural or developing areas (Thomas 1998). In addition to their water supply function, RWHSs are also capable of attenuating floods in some regions (Kumar *et al.* 2005), reducing discharges in the downstream drainage system. For all these reasons, some countries (i.e. UK, Germany and Australia) encourage the use of RWHSs by the introduction of regulations, which make their implementation convenient or, in some cases, mandatory (NSWDP 2005, Eroksuz and Rahman 2010).

The amount of water that can be stored in RWHSs and then used to satisfy residential and industrial water needs strictly depends on the variability of rainfalls. One of the main objectives in the design of RWHSs is the definition of the rainwater storage capacity which maximizes the fulfillment of water demand.

According to several studies, the approaches used in the estimation of the volume of rainwater tanks can be classified in the following groups: i) simplified methods based on user-defined relationships (Ward *et al.* 2010, Palla *et al.* 2011); ii) continuous mass balance simulations (Fewkes and Butler 2000, Liaw and Tsai 2004, Mitchell 2007, Campisano and Modica 2012, 2014); iii) non-parametric approaches based on probability matrix methods (Cowden *et al.* 2008, Basinger *et al.* 2010); and iv) statistical methods (Lee *et al.* 2000, Guo and Baetz 2007, Su *et al.* 2009, Lash *et al.* 2014).

Simplified methods are generally useful for preliminary design but their results should then be tested and verified, due to the potential influence of poor modelling of rainfalls and/or of water storing processes (Ghisi 2010). Typical examples of such methods are the "demand side approach" and the "supply side approach", which do not consider rainfall processes: the design of the tank is only based on the demand of water and the duration of the period of water scarcity in the first case (Fewkes and Butler 2000), and on water availability in the second case. As an alternative, the simplified approach, proposed by Ward *et al.* (2010), is a mix of "demand side" and "supply side approach", and considers both rainfall availability and water demand for the definition of the storage capacity.

Continuous mass-balance simulations are generally more reliable since they consider the complete rainfall process, including its seasonal variations, and the dynamics of water demand. Furthermore, the behaviour of the RWHS can be accurately mimicked and the approach can be easily applied using simple mathematical tools such as spreadsheet applications. The reliability of the results obtained from these approaches is strongly related

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to the use of long-term continuous records of rainfalls (Mitchell 2007) that are often difficult to find or not available at all.

To overcome limitations of both simplified methods and continuous mass-balance simulations, the use of analytical probabilistic approaches has been proposed (Lee *et al.* 2000, Guo and Baetz 2007, Su *et al.* 2009, Lash *et al.* 2014). These methods are based on the analytical derivation of the probability distribution functions for some parameters of interest, starting from the probability distribution function of some rainfall characteristics (Adams and Papa 2000).

In particular, the approaches proposed by Lee *et al.* (2000) and Su *et al.* (2009) are analytically robust and useful at a local scale, but they are difficult to adapt in different locations since the statistical characteristics of the precipitation record are typically hardwired into results. Guo and Baetz (2007) proposed the use of a parametric rainfall simulation, aimed at adapting probabilistic models to different locations, in order to overcome this problem. The main limitation of the parametric simulation is that it considers a maximum of two isolated rainfalls and not the whole chain of events, assuming the tank as being full at the end of the first event. Raimondi and Becciu (2014a, 2014b) have addressed the problem of considering the stochastic process of rainfall, but only to estimate the reliability of a Rain Water Harvesting System. In those papers, the storage capacity is supposed to be known and the Authors considered only the water withdrawal from the tank without any simultaneous refilling during the regulation period.

This paper proposes a further improvement of the method, with an analytical probabilistic approach to estimate the cumulative distribution function of the active storage of rainwater tanks. To test its reliability, an application to a case study is finally presented and the results are compared with those obtained from the continuous simulations of observed data.

2. Probabilistic modelling of a rainwater tank

In most systems for rainwater harvesting, runoff from impervious surfaces, e.g. roofs, is collected into a tank. The estimation of the capacity of this tank is a key issue in the design of RWH systems.

Although tanks have generally to satisfy several storage requirements, in RWH systems the main function is flow equalization. The operation of a tank is essentially a continuous series of cycles in which a filling phase is followed by an emptying one. When the inflow is greater than the required outflow (water surplus periods), the tank stores the excess water volumes; when the opposite occurs (water deficit periods), the tank supplements the inflow with the water volume that was previously stored. The storage reserved for flow equalization is usually called "active storage" or "live storage".

The variation of stored water volume, namely of the part of storage volume used for flow equalization, depends on both inflows and outflows and is different in each cycle. Due to the random nature of both inflow and outflow, this variation can be considered as a random variable obtained as a function of other random variables (inflow and outflow).

Randomness of inflows depends mainly on the natural variability of rainfalls, but may also derive from the estimation uncertainties of both the yielding surface area and the runoff coefficient. In most RWH systems, the yielding surface area A is relatively small and the runoff delay due to surface storage effects is negligible. In each time step, the inflow to the tank I is essentially proportional to rainfall depth h:

$$I = \begin{cases} \varphi \cdot A \cdot (h - h_f) = \varphi \cdot A \cdot h_n & \text{if } h < h_f \\ 0 & \text{if } h \le h_f \end{cases} \tag{1}$$

where φ is the runoff coefficient of the yielding surface and h_f is the so-called "First Flush" (FF) depth, that is the part of rainfall that is usually diverted and not directed to the tank due to the pollution washed off the yielding surface during the first part of rainstorms. The net rainfall depth h_n is defined as the difference between h and h_f, conditioned to the constrain $h > h_f$,

$$h_n = \begin{cases} h - h_f & h > h_f \\ 0 & h \le h_f \end{cases} \tag{2}$$

Most common First Flush devices divert runoff until a fixed volume is reached. To guarantee that only clean water is destined to the tank, the diverter should be emptied between storms even if the fixed volume is not fully reached. Characteristics and amount of runoff pollution depend mainly on the duration of the antecedent dry weather period, the type of yielding surface, the rainfall intensity and duration. The part of runoff that should be intercepted and discarded depends also on the type of the intended use for harvested water.

It has to be observed that the choice of h_f is influenced also by the desired percentage of water that is actually collected. In the systems that have no or limited municipal water backup, for instance, there is the need to discard the minimum quantity of harvested rainfall, accepting also more polluted water to be stored in the tank.

A trade-off between quality and quantity of harvested water has to be found, also considering the costs for water treatment and for supplemental water supply, if available. Standard values of h_f range typically between 0.2 and 2 mm per storm event, corresponding in most cases to a percentage of collected rainwater between 75% and 90% (Doyle 2008).

The net rainfall depth h_n is a non-negative continuous random variable, function of rainfall depth h. The Probability Density Function (PDF) of h_n can be derived from that of h (Benjamin and Cornell 1970):

$$f_{h_n}(x) = f_{h|[h>h_f]}(x + h_f) = \frac{f_h(x + h_f)}{1 - F_h(h_f)} \quad x > 0; h > h_f \tag{3}$$

where F_h is the Cumulative Density Function (CDF) of rainfall depth h. The probability to have a zero net rainfall depth is not null, being possible to have rainfall depths smaller than h_f:

$$Prob\{h_n = 0\} = \mathrm{F}_h(h_f) = \int_0^{h_f} f_h(x)dx \tag{4}$$

The expected value and variance of h_n can be also derived:

$$E[h_n] = \frac{1}{1 - F_h(h_f)} \cdot \left\{ E[h] - \int_0^{h_f} x \cdot f_h(x)dx \right\} - h_f \tag{5}$$

$$VAR[h_n] = \frac{1}{1-F_h(h_f)} \cdot \left\{ VAR[h] + [\mathrm{E}[h]]^2 - \int_0^{h_f} x^2 \cdot f_h(x)dx \right\} - \{E[h_n]\} \tag{6}$$

Also inflow I is a non-negative continuous random variable, function of the random variables φ, A, and h_n. If these three variables are mutually independent, approximated expressions of expected value and variance of I are:

$$E[I] = E[\varphi] \cdot E[\mathrm{A}] \cdot E[\mathrm{h}_n] \tag{7}$$

$$\begin{aligned} VAR[I] &\cong E[\varphi]^2 \cdot E[\mathrm{A}]^2 \cdot VAR\left[\mathrm{h}_n\right] \\ &+E[\mathrm{A}]^2 \cdot E\left[\mathrm{h}_n\right]^2 \cdot VAR[\varphi] + E[\varphi]^2 \cdot E[\mathrm{h}_n]^2 \cdot VAR[\mathrm{A}] \end{aligned} \tag{8}$$

where $E[I]$, $E[\varphi]$, $E[A]$, $E[h_n]$ and $VAR[I]$, $VAR[\varphi]$, $VAR[A]$, $VAR[h_n]$ are respectively the expected values and variances of inflow, runoff coefficient, yielding surface area, and net rainfall depth.

In most cases, the uncertainty of the runoff coefficient φ and of the area of runoff yielding surface A is neglected, both for difficulties of estimation and for the greater relevance of rainfall uncertainty. If φ and A are assumed constant, Equation (8) can be simplified:

$$VAR[I] \cong E[\varphi]^2 \cdot E[\mathrm{A}]^2 \cdot VAR\left[\mathrm{h}_n\right] = \varphi^2 \cdot A^2 \cdot VAR\left[\mathrm{h}_n\right] \tag{8'}$$

Under this hypothesis, the PDF of inflow I can be derived:

$$f_{\mathrm{I}}(x) = \frac{1}{\varphi \cdot A} \cdot f_{h_n}\left(\frac{x}{\varphi \cdot A}\right) = \frac{1}{\varphi \cdot A} \cdot \frac{f_h\left(\frac{x+h_f}{\varphi \cdot A}\right)}{1 - F_h\left(h_f\right)} \quad x > 0; h > -\frac{h_f}{\varphi \cdot A} \tag{9}$$

The probability to have a zero inflow is not null and is equal to the probability to have a zero net rainfall depth (see Equation (4)):

$$Prob\{I = 0\} = Prob\left\{h_n = 0\right\} = \mathrm{F}_h\left(h_f\right) = \int_0^{h_f} f_h(x)dx \tag{10}$$

The above equations can be used to estimate probabilities of inflows from statistical analysis of rainfalls on event-scale. Rainfall records, however, are often available only on daily or monthly scales. Although equations are still valid also for these scales, some considerations on the FF depth h_f should be done.

When daily records are used, it is possible to merge two or more rainfall events in the same day, depending on the Inter-Event Time Definition (IETD), defined as the minimum dry time that is assumed to be necessary to consider independent two consecutive storms. Then, subtraction of the FF depth h_f on daily scale leads to values of net rainfall depths that are on average smaller than the ones obtained on event-scale. On the other hand, some storms have duration longer than a day, leading to an opposite effect.

The two effects tend to balance each other out, although one may prevail depending on the climatic context. The numerical relevance of the combined effect depends also on the value of h_f. In the case study of Milano (Italy), an IETD of 1 hour was used in our study and the simulation on daily scale leads to an overestimation of the annual inflow, in comparison to the analysis on event-scale, ranging from about 1% for $h_f = 0.2$ mm to about 6% for $h_f = 2$ mm.

When only monthly records are available, a major issue is the estimation of the monthly value of FF depth to be applied. Some events have a rainfall depth lower than h_f and the cumulated FF abstraction in a month is then smaller than the product of h_f and the mean monthly number of events. This is particularly true when there is the preponderance of short events, with possible multiple occurrences in the same day. In this case, application of FF abstraction to daily rainfall depths leads to a partial balance. An approximate solution is then to consider the product of h_f and the mean monthly number of rainy days. In the case study of Milano (Italy) this number is equal to about 7. Considering a FF depth on event-scale in the range $h_f = 0.2$–2.0 mm, the monthly FF depth may be in the range 1.4–14 mm.

Under the hypothesis that the number of rainfall events in a fixed period is independent of rainfall depths, it is possible to derive the mean number M_{ne} of rainfall events with inflow to the tank, that is the mean number of events with a rainfall depth h greater than h_f:

$$M_{ne} = M_e \cdot Prob\left\{h > h_f\right\} = M_e \cdot \left[1 - F_h(h_f)\right] \tag{11}$$

when M_e is the mean number of rainfall events in the same period.

Water demand varies according mainly to the type of use, but its estimation is uncertain due to lack of knowledge about real consumption and it should be considered a random variable too (Kellagher ~~2012~~). Indoor domestic use, for instance, varies according to the (random) number of users, to their life habits, to seasonal weather changes, etc. Actual availability and supply costs also influence water consumption (Ghisi 2010). Jorgensen and Graymore (2009) argue that also trust, both at the inter-personal and institutional level, plays a role in household water consumption.

While mean daily water requirements per capita are easily available for different climatic and socio-economic contexts (see e.g., AWWA 1999, Nauges and Whittington 2010), the estimation of their variability is more difficult due to the number of factors involved. For this reason, in this study a constant water demand is assumed.

3. Design of a rainwater tank

The active storage of a rainwater tank should be enough to satisfy, by flow equalisation, the complete fulfilment of water demand. In other words, the active storage of a tank is based on the maximum storage volume required to take account of the maximum difference in the volume between inflows and outflows over a specific period of time before recovering to the same initial state; this implies that the yield in the considered sub-period must be greater than the demand. To estimate the active storage volume, the maximum variations of stored water volumes have to be analysed in a defined period of time.

Although both hydrological phenomena and water demand have a main cycle of one year, flow equalisation is usually required on smaller time scales. The typical aim of these tanks is effectively to supplement water during seasonal periods of rainwater scarcity, in most cases not longer than few months. Often, also for stored water quality issues, one month is chosen as the characteristic period for flow equalisation.

The operation of the tank, then, can be considered as a series of flow balancing periods of length T_B, each of which is associated to a different variation of stored water volume. These periods are not really independent, as water deficits may last for more than one period. The effect of this correlation is that storage conditions at the beginning and at the end of each period are not the same.

However, it has to be noted that usually the balancing period is precisely chosen considering the average duration of rainwater scarcity periods. The probability of water deficits that are significantly longer than this duration is then low. So, the variations of stored water volume in each balancing period can be assumed to be approximately independent.

The maximum value of this variation is positive for surplus periods and negative for deficit periods. Positive variations can be theoretically unlimited on the upper part, depending mainly on rainfall. Negative ones, cannot be greater in modulus than the total water demand in the flow balancing time period. Therefore, in each surplus period there is no need to store more water than the total water demand in the successive deficit period, unless a carry-over from one cycle to the other is requested.

For this reason, a simple and common way to design the tank is to define the active storage volume equal to the total water demand in the ~~equalisation~~ characteristic period. This is called demand-side approach.

This approach, however useful for a first estimate, may lead to an overestimation of the active storage. In most cases, there is some inflow even in deficit periods. Moreover, inflows may not always be enough to completely fill this storage, especially with significant water demands.

It is important to remark that, owing to the random nature of both inflows and outflows, the complete fulfilment of water demands cannot be guaranteed, whatever is the active storage of the tank, but only associated to a probability level.

A more proper approach is to estimate the active storage volume W_B as the maximum difference between the cumulated outflow D_t and the cumulated inflow I_t at time t in a deficit period, that is given the condition $D_B > I_B$:

$$W_B = \max_{0 \le t \le T_B} \left(D_t - I_t | D_B > I_B\right) \tag{12}$$

where D_B and I_B the tank's cumulated outflow and inflow at the end of the period T_B.

Although W_B is a random variable, generally increasing with the length T_B of the balancing period, it is limited at its extremes. The minimum value is obviously $W_{Bmin} = 0$: when the cumulated inflow I_t is greater than or equal to the cumulated water demand D_t for all the period T_B. The maximum value is, as already remarked, equal to the total water demand in the period $W_{Bmax} = D_B$ and is reached when $I_B = 0$.

Analyzing the inflow process at daily scale, and assuming that the daily values of h, h_n and I are statistically independent, the probability of this condition is:

$$\begin{aligned}
\text{Prob}\left\{W_B = W_{Bmax}\right\} &= \text{Prob}\left\{W_B = D_B = E[D_t] \cdot N_B\right\} = \text{Prob}\left\{I_B = 0\right\} \\
&= p_{N_{rd}}(0) + \sum_{n=1}^{N_B} p_{N_{rd}}(n) \cdot \left[\text{Prob}\{I = 0\}\right]^n \\
&= p_{N_{rd}}(0) + \sum_{n=1}^{N_B} p_{N_{rd}}(n) \cdot \left[F_h(\text{h}_f)\right]^n
\end{aligned} \tag{13}$$

where $E[D_t]$ is the mean value of the water demand, N_B is the total number of days and $p_{N_{rd}}(x)$ is the probability distribution of the number N_{rd} of rainy days, all referred to a time period of length T_B. It is often assumed that rainfall events follow a Poisson stochastic process (Rodriguez-Iturbe *et al.* 1987), so the distribution of N_{rd} is:

$$p_{N_{rd}}(n) = \frac{M_{rd}^n \cdot e^{-M_{rd}}}{n!} \tag{14}$$

where M_{rd} is the mean number of rainy days in a time period of length T_B. The rainy days with a non-zero inflow can be assumed also to follow a Poisson stochastic process, so the distribution of its number N_{nrd} in T_B is:

$$p_{N_{nrd}}(n) = \frac{M_{nrd}^n \cdot e^{-M_{nrd}}}{n!} \tag{15}$$

where M_{nrd} is the mean number of rainy days with non-zero inflow in a time period of length T_B. The two numbers are linked by the relationship (see also Equation (11)):

$$M_{nrd} = M_{rd} \cdot \left[1 - F_h(h_f)\right] \tag{16}$$

It is worthwhile to note that, from Equation (14) the following approximation of Equation (13) can be derived if N_B is high enough:

$$\begin{aligned}
&\text{Prob}\left\{W_B = W_{Bmax}\right\} \\
&\approx \text{e}^{-M_{rd} \cdot \left[1 - F_h(h_f)\right]} = \text{e}^{-M_{nrd}} = p_{N_{nrd}}(0) = Prob\left\{I_B = 0\right\}
\end{aligned} \tag{17}$$

The same relationship can be derived exactly if Equations (15) and (16) are also considered.

In the cases with $I_B > 0$, W_B is smaller than D_B. Its estimation is not straightforward, due to the difficulty of a mathematical representation of the two functions D_t and I_t. A simplified approach, however, can be followed.

First, water demand is assumed constant in time and equal to its mean value in T_B: $D_t = \text{E}[D_t] = D$. With this assumption, the variations of the daily water demand in comparison to its mean, for example among different days in the week, are obviously neglected. Variations inside the day are also implicitly ignored, due to the chosen daily scale of analysis.

Second, it is assumed that W_B occurs at the end of a critical deficit sub-period of length $T_C \le T_B$, consisting of a row of "dry" days, namely without inflow to the tank, followed by a row of "wet" days, namely with inflow to the tank. The definition of deficit sub-period implies, of course, that the condition $I_B \le D_C = D \cdot N_C \le D_B$ must hold, being N_C the number of days in T_C.

Expressing the cumulated inflow at the end of the deficit period, namely at time T_B, as:

$$I_B = \varphi \cdot \text{A} \cdot \sum_{i=1}^{N_{nrd}} h_{n_i} = \varphi \cdot A \cdot H_n \tag{18}$$

the mean value of N_C in T_C is expressed as the sum of the mean number of "dry" days and the mean number of "wet" days with inflow smaller than D_B:

$$M_C = (N_B - M_{nrd}) + M_{nrd} \cdot F_{I_B}\left(D_B\right) = N_B - M_{ne} \cdot \left[1 - F_{H_n}\left(\frac{D_B}{\varphi \cdot A}\right)\right] \tag{19}$$

The active storage can then be assumed approximately equal to the difference $W_B = D \cdot M_C - I_B$. The cumulative distribution function of W_B can be derived, under the above cited hypotheses:

Table 1. Main statistics of daily rainfall data at monthly scale ($h_f = 0$). Values in the last column are related to the average month in the year.

	J	F	M	A	M	J	J	A	S	O	N	D	Year
$E[h_G]$ [mm]	10.50	13.21	10.98	9.71	10.19	9.06	11.33	13.43	13.96	13.43	11.98	13.13	11.65
$VAR[h_G]$ [mm]	51.22	142.74	63.50	17.31	43.28	25.47	61.88	76.74	66.44	47.34	60.42	223.75	71.88
$E[N_{rd}]$=Me [days]	6.17	5.31	6.21	8.94	9.82	7.82	5.85	6.59	6.61	7.53	7.55	5.94	7.03
$VAR[N_{rd}]$[days]	13.85	13.64	10.11	18.93	16.15	7.90	6.37	9.52	11.05	14.07	15.51	11.22	13.45

$$F_{W_B}(x) = \text{Prob}\left\{D \cdot M_C - I_B \le x | D \cdot M_C \ge I_B\right\}$$
$$= \text{Prob}\left\{H_n \ge \frac{D \cdot M_C - x}{\varphi \cdot A} | H_n \le \frac{D \cdot M_C}{\varphi \cdot A}\right\} \quad (20)$$
$$= \frac{F_{Hn}\left(\frac{D \cdot M_C}{\varphi \cdot A}\right) - F_{Hn}\left(\frac{D \cdot M_C - x}{\varphi \cdot A}\right)}{F_{Hn}\left(\frac{D \cdot M_C}{\varphi \cdot A}\right)} = 1 - \frac{F_{Hn}\left(\frac{D \cdot M_C - x}{\varphi \cdot A}\right)}{F_{Hn}\left(\frac{D \cdot M_C}{\varphi \cdot A}\right)}$$

The function H_n is the cumulated net rainfall depth in the deficit period of length T_B, namely the sum of a random number N_{nrd} of independent random variables h_{ni}. Although the derivation of its CDF is not straightforward, the estimation of its main moments is easier.

Calculation of the mean value of H_n, can be performed in two steps, taking the expectation first in relation to the net rainfall depths (symbol E with subscript h) and then in relation to the number of net rainfall events (symbol E with subscript N) (Benjamin and Cornell 1970). Assuming a fixed number $N_{nrd} = N$ of days with positive inflow, the first expectation is that:

$$E_h\left[H_n\right] = \left[\sum_{i=1}^{N_{nrd}} h_{n_i} | N_{nrd} = N\right] = N \cdot E[h_n] \quad (21)$$

The second expectation, in order to consider the random nature of N_{nrd}, is that:

$$E_N\left[H_n\right] = E_N\left[E_h\left[H_n | N_{nrd} = N\right]\right] = E_N\left[N \cdot E[h_n]\right] = M_{nrd} \cdot E[h_n] \quad (22)$$

The calculation of the variance can be performed in the same way, using the well-known relationship:

$$VAR\left[H_n\right] = E\left[H_n^2\right] - E^2\left[H_n\right] \quad (23)$$

The first expectation in Equation (23) can be taken in two steps too, with the same meaning of subscripts used above. Using Equations (21) and (22) and remembering that the variance of a sum of independent random variables is the product of the variances, the result is:

$$\begin{aligned} E\left[H_n^2\right] &= E_N\left[E_h\left[H_n^2 | N_{nrd} = N\right]\right] \\ &= E_N\left[VAR_h\left[H_n | N_{nrd} = N\right] + E^2\left[H_n | N_{nrd} = N\right]\right] \\ &= E_N\left[N \cdot VAR\left[h_n\right] + N^2 \cdot E^2[h_n]\right] \\ &= M_{nrd} \cdot VAR\left[h_n\right] + E\left[N_{nrd}^2\right] \cdot E^2\left[h_n\right] \end{aligned} \quad (24)$$

Merging Equations (22), (23) and (24), the final result is:

$$\begin{aligned} VAR\left[H_n\right] &= M_{nrd} \cdot VAR\left[h_n\right] + E^2\left[h_n\right] \cdot \left\{E\left[N_{nrd}^2\right] - M_{nrd}^2\right\} \\ &= M_{nrd} \cdot VAR\left[h_n\right] + E^2\left[h_n\right] \cdot \mathrm{VAR}\left[N_{nrd}\right] \end{aligned} \quad (25)$$

For the mean $E[h_n]$ and the variance $VAR[h_n]$ of net rainfall depth h_n, see Equations (5) and (6). Moreover, it is useful to note that, assuming a Poisson distribution for N_{nrd}, its variance is equal to its mean. So:

$$VAR\left[N_{nrd}\right] = E\left[N_{nrd}\right] = M_{nrd} \quad (26)$$

4. Case study

A hypothetical rainwater harvesting system in Milano, Italy, was considered, with a completely impermeable roof (φ = 1.0) of area A equal to 1000 m^2 and a storage tank. Two approaches for the estimation of the tank's active storage were compared, continuous simulation and semi-probabilistic derivation (Equation (20)). A flow equalization period of one month (T_B = 1 month) was assumed.

Four different daily FF abstractions (h_f = 0, 0.5, 1.0 and 2 mm) and three constant rates of water demand (D = 0.01, 0.05, 0.1 m^3/hour) were considered. Assuming an average need for non-potable water of 48 liters/day per person, this corresponds to a total water demand of 5, 25, and 50 persons respectively.

A series of continuous rainfall depths from the Via Monviso gauge station, with an original time resolution of one minute in the period 1971–2005, was used. A total of 4336 independent rainfall events were identified using an IETD = 1 hour. Daily and monthly rainfall depths were calculated. The main statistics of the daily rainfall data, at annual and monthly scale, have been reported in (Table 1).

A chi-squared test was performed to verify if the monthly number of non-zero inflows N_{nrd} follows the Poisson probability distribution. The hypothesis was tested for all the four values assumed for the daily FF abstraction, namely for h_f = 0, 0.5, 1.0, 2.0 mm. In the first case (h_f = 0) the number of non-zero inflows N_{nrd} obviously corresponds to the number of rainy days, namely $N_{nrd} = N_{rd}$.

The hypothesis of Poisson distribution for N_{nrd} cannot be confirmed when the whole year is considered, due to seasonal effects. The test was then performed on a shorter sub-period. The three months of June, July and August were chosen as critical for the RWH system. Although this sub-period is only the second most critical in terms of mean total rainfall in Milano, it can be considered the most critical for the RWH system, being water demands higher during summer months.

However, the same hypothesis was not rejected at the 5% confidence level, in all the four cases of FF abstraction, when the analysis is restricted to the three months from June to August. Using 11 classes for a sample of 105 months, the values of the calculated χ^2 statistic range from 14.394 (for h_f = 0) to 9.430 (for h_f = 2.0), with a critical value of 16.919 (5% confidence level; 11 - 2 = 9 degrees of freedom, taking into account that the distribution parameter was estimated from the sample). The observed and the Poisson expected CDF of N_{nrd} are reported, as an example, in Figure 1.

The operation of the tank was simulated at the daily scale, in order to calculate the maximum positive difference between the cumulated outflow and the cumulated inflow in each month in the period 1971–2005. The simulation was performed according to a Yield Before Spillage (YBS) operating rule (Fewkes and Butler 2000). Deficit periods between two consecutive months were considered.

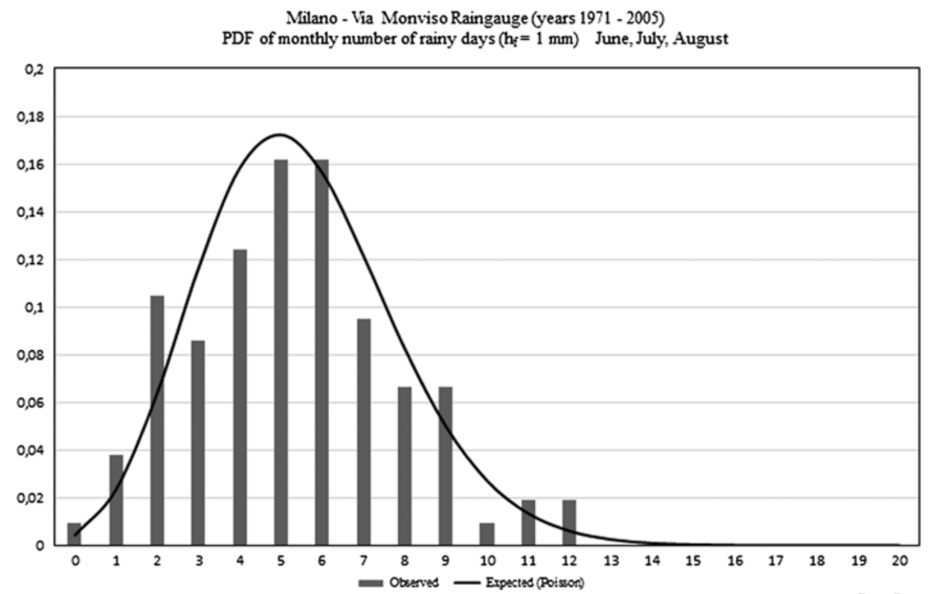

Figure 1. Observed (dashed line) and expected Poisson (solid line) PDF of the number N_{nrd} of days with non-zero inflow (case with $h_f = 1.0$ mm).

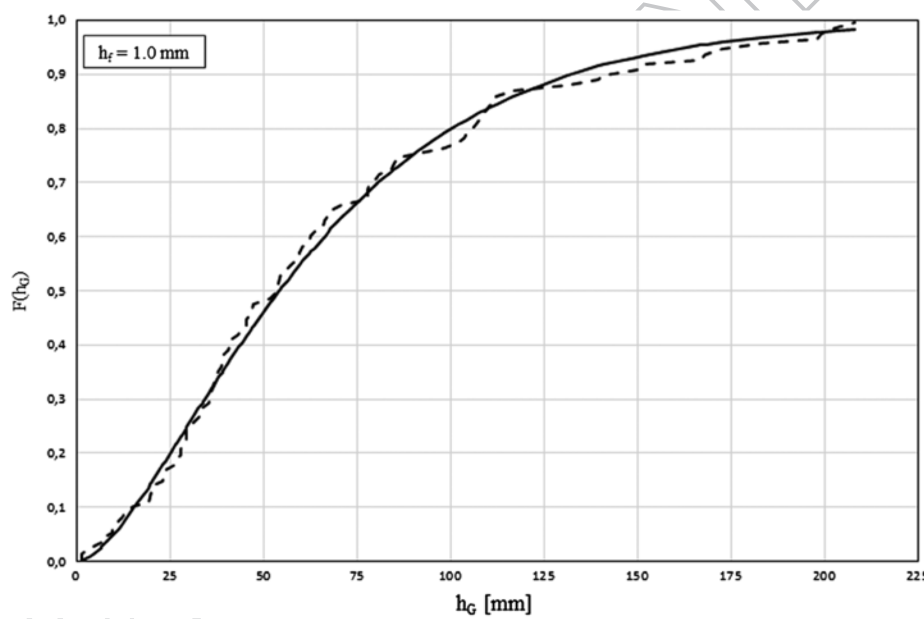

Figure 2. Observed (dashed line) and fitted Gamma CDF of the cumulated monthly net rainfall H_n (case with $h_f = 1.0$ mm).

These maxima represent a sample of the monthly active storage of the tank. The values of the chosen sub-period of analysis, namely the three summer months, were extracted from the original sample. The Gringorten plotting position (Chow *et al.* 1988) was used to estimate their sample cumulative frequency function. For the same months, daily rainfall statistics were used to estimate the CDF of the active storage of the tank from Equation (20). A 2-parameter Gamma distribution was assumed for the monthly net rainfall depth $H_{N.}$ Parameters were estimated by the method of moments (Benjamin and Cornell 1970). The observed and the 2-parameter Gamma CDF of H_n are reported for the case $h_f = 1.0$ mm, as an example, in Figure 2.

In Figures 3a,3b,3c,3d,3e,3–f, the sample cumulative frequency functions of the monthly active storage are compared with the theoretical CDFs from equation (20) for different combinations of FF depth h_f and daily water demand D.

As can be seen, the proposed procedure seems to provide a good estimation of CDF of active storage for the case study. Although some simplifying hypotheses were made, some of which were tested, namely on the distribution of the number of rainy days and of the cumulated net rainfall depths, results seem comparable to those obtained by continuous simulation. A chi-squared test was performed to verify the goodness-of-fit of the estimated PDF. As an example, in the case with $D = 0.1$ m^3/h and $h_f = 0$ mm, the obtained χ^2 value was 6.03, compared to the critical value 11.07 (for a 5% confidence level, with five degrees of freedom).

The fitness of the theoretical CDF, estimated by equation (20), seems to increase with h_f and this may be explained by

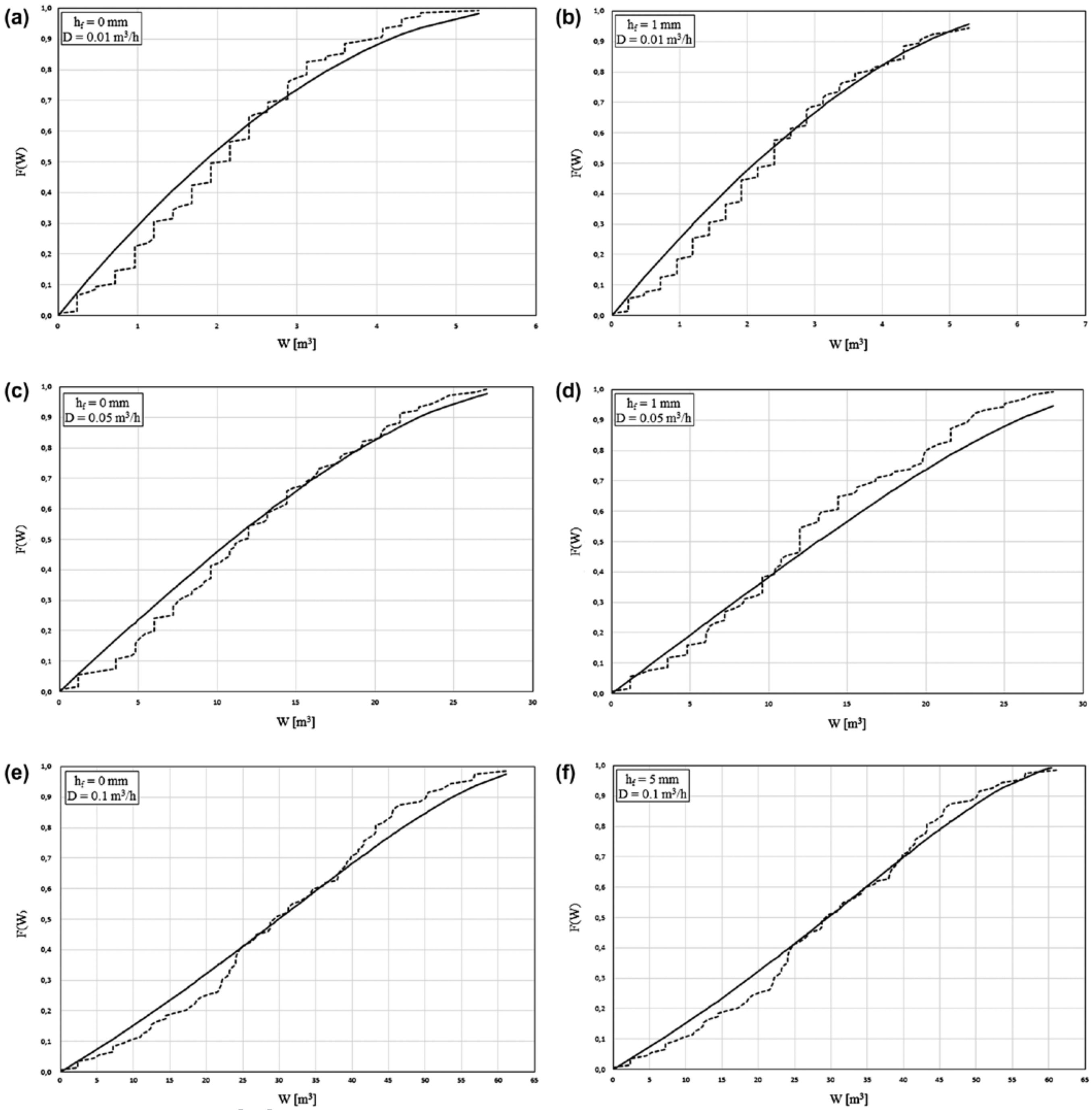

Figure 3. Observed (dashed line) and Equation (20) (solid line) CDF of the active storage volume W_B of the tank.

the reduction of the length of non-zero inflow periods. A more detailed analysis of this issue is currently in progress.

Conclusions

A probabilistic approach for the operation analysis of tanks in RWHSs was suggested, in order to overcome limitations of simplified methods. Relationships for the capacity design were developed and proposed, as an alternative to continuous simulations.

Equation (20) can also be used in the common case in which long series of rainfall records are not available for continuous simulation. Using the proposed equations, the probabilistic estimation of the active storage of the tank is possible only if the main moments, expected value and variance, of daily or monthly rainfall depths and of rainy days are available.

An application to the case study was performed and results are in good agreement with those from continuous simulation. A preliminary positive evaluation of the procedure applicability can then be considered, although more tests with rainfall series recorded in different climatic zones should be performed in the future.

List of symbols

A: yielding surface area
φ: runoff coefficient of the yielding surface
E[x]: expected value of *x*

$VAR[x]$: variance of x
f_x: Probability Density Function of x
F_x: Cumulative Density Function of x
h: rainfall depth
h_f: First Flush depth
h_n: net rainfall depth
h_G: daily rainfall depth
H_n: cumulated net rainfall depths in T_B
T_B: length of the flow balancing period
$E_h[H_n]$: expected value of H_n with respect to rainfall depth h
$E_N[H_n]$: expected value of H_n with respect to the number of net rainfall events
I: inflow to the tank
I_t: cumulated inflow to the tank at time t in T_B
I_B: cumulated inflow at the end of the period T_B
T_C: duration of the critical deficit sub-period
N_C: number of days in T_C
$M_C = E[N_C]$: expected value of N_C
D_t: cumulated water demand (cumulated outflow) at time t in T_B
D_B: cumulated water demand at the end of the period T_B
D_C: cumulated water demand at the end of the period T_C
$D = E[D_t]$: expected value of D_t
N_B: total number of days in T_B
N_{rd}: number of rainy days in T_B
M_{rd}: mean number of rainy days in T_B
$p_{Nrd}(x)$: probability distribution of N_{rd}
N_{nrd}: number of rainy days with a non-zero inflow in T_B
$M_{nrd} = E[N_{nrd}]$: expected value of N_{nrd}
$p_{Nnrd}(x)$: probability distribution of N_{nrd}
M_e: mean number of rainfall events
M_{ne}: mean number of rainfall events with inflow to the tank
W_B: active storage volume in T_B
W_{Bmin}: minimum active storage volume in T_B
W_{Bmax}: maximum active storage volume in T_B
$F_{WB}(x)$: Cumulative Distribution Function of W_B

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